

# Robustification of the generalized predictive law (GPC) by the implicit application of the $H_\infty$ method

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## ABSTRACT

In this paper, we propose a method of robustification of the initial polynomial form of the generalized predictive law (GPC) for a permanent magnet synchronous machine (PMSM). For this, the procedure consists of the following three steps: First, synthesis of an initial predictive controller to ensure a better follow-up of the properties of the closed-loop system. Secondly, a robust  $H_\infty$  controller is synthesized by solving the mixed sensitivity problem by using the two Riccati equations to ensure a better dynamics in regulation. Third, the two previous controllers are combined, using Youla's parameterization to determine a robustified GPC controller. This controller should simultaneously satisfy the same better tracking dynamics of the initial GPC predictive controller. In addition, it should provide the same robustness of the robust  $H_\infty$  controller. To validate the efficiency of this method, a permanent magnet synchronous machine (PMSM), which presents a real process, The dynamic behavior of the proposed process is modeled by an uncertain model. In our case the nominal model is used for the synthesis of the GPC controller with and without noise, thus used for the synthesis of the controller  $H_\infty$ . The system is controlled by the three previous controllers where their results are compared in the time and frequency domains.

## KEYWORDS

GPC Controller, Robust Controller, Mixed Sensitivity Problem, Permanent Magnet Synchronous Machine.

## 1 INTRODUCTION

Generalized predictive control [1] is a very powerful method. It has been the subject of much research over the last decade. In the control of MIMO systems, it is generally suggested to use the CARIMA model (Integrated Controlled, Regressive, Auto-Regressive) with the GPC [2], in which the expected behavior of the actual process can be predicts in an extended time horizon. The GPC law derives from the iterative resolution of two Diophant equations and the minimization of the quadratic criterion at each step [3]; [4].

The GPC law can be transformed into an equivalent RST polynomial form. This transformation is highly desired in control engineering to reduce the recursive resolution of Diophant's equations and the ability to examine the stability and robustness of the performances, where the singular maximum values of its sensitivity and its complementary sensitivity functions are used in the frequency domain.

Several works have been developed to control a nominal plant using GPC controllers, where a good tracking dynamic of the set point reference is guaranteed. Unfortunately, its stability and performance robustness are not ensured in a set of neighbouring plant cases [5]; [6]. Various design synthesis methods have been proposed also since the late of the nineties in order to resolve this problem. The C-polynomial of the CARIMA model to enhance the dynamic of the load disturbance inputs rejection is used by [7]. Unfortunately, this choice remains complicated for plants that are given by a transfer matrix of higher order. An additional parameter proposed called Q-parameter [5]; [6]; [8]; [9]. This parameter is introduced in order to achieve a good compromise between both the stability and the performances robustness.

Stoica et al., (2008) propose a robustification method based on a constrained linear optimization where the designed controller is based on two-step procedures [10]. An initial GPC controller is first designed with a deterministic model, and its robustness is then enhanced via the Youla parameterization. This parameterization allows formulating frequency and time domain constraints as a convex optimization problem. Afterwards, this problem is approximated by a linear programming with inequality constraints, and the optimal Q-parameter is derived. A disadvantage of this method is expressed by a hard choice of an optimal transfer matrix order of the Q-parameter which was determined after several trials. In addition, the optimal solution of the optimization search problem is not guaranteed. The convergence speed gets slowing down when approaching the optimal solution due to the higher number of the Q-parameter variables. These variables are determined by optimization and they depend on the plant dimension. Consequently, the computational cost increases exponentially as the optimization size increases and becomes rapidly prohibitive as the order of the Q-parameter increases, which leads to numerical ill-conditioning.

This method requires also a good choice of the Q-parameter order which is very difficult to find and would be given after several trials.

Knowing that, the  $H_\infty$  control theory offers the possibility of including robustness considerations explicitly in the synthesis controller step [11]. The controller synthesis problem can be formulated as a LMIs optimization problem. Several works have shown the efficiency of the  $H_\infty$  controller to control an uncertain plant. Its stability and performance robustness are guaranteed with better margins. Unfortunately, the obtained tracking dynamic of the set-point reference is degraded when their robustness margins are increased. In this paper, we propose a ruggedized GPC controller that assures the robustness of the  $H_\infty$  controller while maintaining the GPC tracking dynamics preserved. The fundamental difference between the robustification method proposed in this paper and those available in the literature is how to determine the components of the Q parameter transfer matrix.

## 2 DESIGN OF THE GPC LAW

The GPC algorithm consists of applying a control sequence that minimizes a multistage cost function of the form [3]; [12],

$$J = \sum_{k=N_1}^{N_1+N_2-1} \|\hat{y}(t+k/t) - w(t+k)\|_\Psi^2 + \sum_{k=1}^N \|\Delta u(t+k-1)\|_\lambda^2 \quad (1)$$

Where  $N_1$  and  $N_2$  are the minimum and maximum costing horizons,  $\Psi$  and  $\lambda$  are the positive definite weighting matrices that penalize the errors and the control actions respectively,  $N_u$  is the control horizon,  $w(t+k)$  is a future set-point or reference sequence and  $\hat{y}(t+k/t)$  is the optimum  $k$ -step ahead prediction of the system output on data up to time  $t$ .

In the MIMO case, the CARIMA model with  $(m \times n)$  inputs-outputs is defined by [3]; [12],

$$\Delta(q^{-1})A(q^{-1})y(t) = \Delta(q^{-1})B(q^{-1})u(t-1) + C(q^{-1})\zeta(t) \quad (2)$$

Where  $y(t) = [y_1(t), y_2(t), \dots, y_m(t)]^T$ ,

$$u(t) = [u_1(t) \quad u_2(t) \quad \dots \quad u_n(t)]^T,$$

$$\zeta \in \Re^{m \times 1} = [\zeta_1 \quad \zeta_2 \quad \dots \quad \zeta_m]^T$$

and  $\Delta(q^{-1}) = (1 - q^{-1})I_{m \times m}$

$A(q^{-1})$ ,  $B(q^{-1})$  and  $C(q^{-1})$  are polynomials in backward shift operator  $q^{-1}$  that respectively defined by:

$$A(q^{-1}) = I_{m \times m} + \sum_{j=1}^{j=n_a} A_j q^{-j}, \quad B(q^{-1}) = \sum_{j=0}^{j=n_b} B_j q^{-j},$$

$$C(q^{-1}) = \sum_{j=0}^{j=n_c} C_j q^{-j} \quad (3)$$

$C(q^{-1})$  is equal to the unity (i.e.  $C(q^{-1}) = I_{m \times m}$ ). The optimum  $k$ -step ahead prediction over the costing horizon  $1 \leq k \leq N_2$  is given by

$$\hat{y}(t+k/t) = H_k(q^{-1})\Delta u(t-1) + F_k(q^{-1})y(t) + G_k\Delta u(t+k-1) \quad (4)$$

Where,  $H_k$ ,  $F_k$  and  $G_k$  are the polynomial matrices in the backward shift operator  $q^{-1}$  that determined from solving iteratively Diophantine equations.

## 3 DESIGN OF THE INITIAL GPC CONTROLLER

In the GPC algorithm, the receding horizon principle assume that only the  $(n)$  first component of the optimal control sequences resulting from the minimization of (1) are applied, so that at the next sampling time the same procedure is repeated. The obtained control law can be transformed to a RST structure that given as [1]; [5]; [12]

$$S(q^{-1})\Delta(q^{-1})u(t) = T(q^{-1})w(t) - R(q^{-1})y(t) \quad (5)$$

$S(q^{-1})$ ,  $R(q^{-1})$  and  $T(q^{-1})$  are respectively defined as

$$S(q^{-1}) = I_{n \times n} + q^{-1}M_1(q^{-1})H(q^{-1})$$

$$R(q^{-1}) = M_1(q^{-1})F(q^{-1}) \quad (6)$$

$$T(q^{-1}) = M_1(q^{-1})[q.I_{m \times m} \quad \dots \quad q^{N_2}.I_{m \times m}]^T$$

Where  $F$  and  $H$  are the polynomial matrices that defined by  $H = [H_1, \dots, H_{N_2}]^T$ ,  $F = [F_1, \dots, F_{N_2}]^T$  respectively.  $M_1(q^{-1})$  is the  $(n)$  first component of the polynomial matrix  $(G^T \Psi G + \lambda)^{-1} G^T$ .

Noticed that the better reference tracking performances of the initial GPC controller depends heavily by a good choice of the tuning parameters of the GPC law which are:  $N_2$ ,  $N_u$ ,  $\Psi$  and  $\lambda$ .

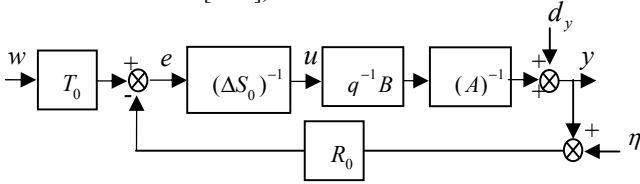
More details for choosing these parameters are available in references [13] and [14].

## 4 YOULA PARAMETERIZATION

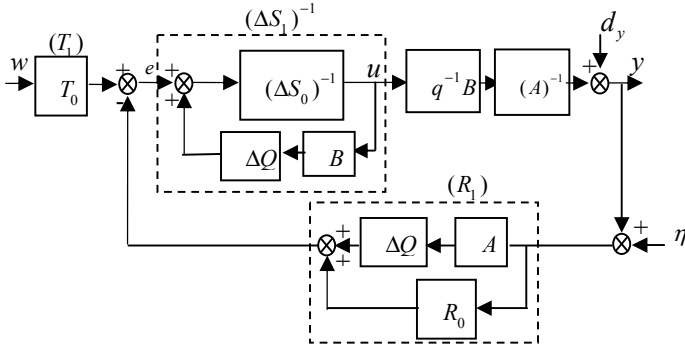
In the next section, we assumed that the robustified *GPC* controller is presented by the polynomial matrices  $R_1 S_1 T_1$ . However, the initial one is presented by the polynomial matrices  $R_0 S_0 T_0$ . The proposed Youla parameterization is defined as [4], [15],

$$\begin{aligned} R_1(q^{-1}) &= R_0(q^{-1}) + \Delta(q^{-1})Q(q^{-1})A(q^{-1}) \\ S_1(q^{-1}) &= S_0(q^{-1}) - q^{-1}Q(q^{-1})B(q^{-1}) \\ T_1(q^{-1}) &= T_0(q^{-1}) \end{aligned} \quad (7)$$

Where  $Q(q^{-1})$  is the stable polynomial matrix to be determined. The feedback control systems before and after parameterization are shown as follows [9-13],

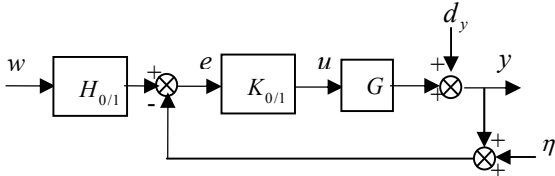


**Figure 1. Feedback control system based on initial 2-DOF-MGPC controller**



**Figure 2. Feedback control system based on robustified 2-DOF-MGPC controller**

Moreover, both previous diagrams are presented by the standard feedback control system that given as [16],



**Figure 3. Standard Feedback control system**

Where,  $e(t)$ ,  $d_y(t)$  and  $\eta(t)$  are respectively the errors, the disturbances and the measurement noises.  $G(q^{-1})$ ,  $H_0(q^{-1})$ ,  $K_0(q^{-1})$ ,  $H_1(q^{-1})$  and  $K_1(q^{-1})$  are polynomial matrices that respectively defined by

$$\begin{aligned} G &= q^{-1} A^{-1}(q^{-1}) B(q^{-1}) \quad \begin{cases} K_0 = [\Delta(q^{-1}) S_0(q^{-1})]^{-1} R_0(q^{-1}) \\ H_0 = R_0^{-1}(q^{-1}) T_0(q^{-1}) \end{cases} \\ \begin{cases} K_1 = [\Delta(q^{-1}) S_1(q^{-1})]^{-1} R_1(q^{-1}) \\ H_1 = R_1^{-1}(q^{-1}) T_1(q^{-1}) \end{cases} \end{aligned} \quad (8)$$

According to figure 3, the output of the feedback control system can be expressed as

$$y(t) = G_{cl}(q^{-1})w(t) + S_d(q^{-1})d_y(t) - S_c(q^{-1})\eta(t) \quad (9)$$

Where  $G_{cl}(q^{-1})$  is the polynomial matrix that gives information on the tracking performances.  $S_{d_0}(q^{-1})$  denotes the direct sensitivity matrix that provides the information on the *NP* against the load disturbances in low frequency range [17-19].  $S_{c_0}(q^{-1})$  is the complementary sensitivity matrix that gives the information on the *RS* toward the neglected dynamics and the effect of the measurement noises in high frequency range [17-19]. These polynomial matrices are defined by

$$\begin{aligned} G_{cl_0}(q^{-1}) &= G K_{0/1} (I_{m \times m} + G K_{0/1})^{-1} H_{0/1} \\ S_{d_0}(q^{-1}) &= (I_{m \times m} + G K_{0/1})^{-1} \\ S_{c_0}(q^{-1}) &= G K_{0/1} (I_{m \times m} + G K_{0/1})^{-1} \end{aligned} \quad (10)$$

The new complementary sensitivity matrix then defined as

$$\begin{cases} S_{c_0} = (R_0 + q \Delta S_0 B^{-1} A)^{-1} R_0 \\ S_{c_1}(q^{-1}) = S_{c_0}(q^{-1}) [I_{m \times m} + Q_f(q^{-1})] \end{cases} \quad (11)$$

Finally, the modified direct sensitivity matrix of the robustified *GPC* controller is defined by

$$\begin{cases} S_{d_0} = (R_0 + q \Delta S_0 B^{-1} A)^{-1} q \Delta S_0 B^{-1} A \\ S_{d_1} = S_{d_0} (I_{m \times m} - q^{-1} A^{-1} B [\Delta S_0]^{-1} R R^{-1} \Delta Q A) \end{cases} \quad (12)$$

Knowing that, the sum of the initial sensitivities that provided by the primary *GPC* controller is defined as

$$S_{c_0}(q^{-1}) + S_{d_0}(q^{-1}) = I_{m \times m} \Rightarrow S_{c_1}(q^{-1}) + S_{d_1}(q^{-1}) = I_{m \times m} \quad (13)$$

the same tracking performance (i.e.  $G_{cl_i} = G_{cl_0}$ ) by the robustified *GPC* controller. The above equation yields also,

$G_{cl_i}(q^{-1}) = G_{cl_0}(q^{-1})$ . We notice that, a perfect template of direct sensitivity matrix is achieved when their maximal singular values are vanishing as much as possible in low-frequencies and approach to unity in high-frequencies[20]; [21]. On the other hand, the ideal form of the complementary sensitivity matrix is achieved when their maximal singular values are vanishing as much as possible in high-frequencies and approach to unity in low-frequencies[20], [21]. Consequently, according to (7), (8) and (10), the robustness proprieties given by the initial *GPC* controller can be improved using an optimal *Q*-parameter whereas their reference tracking proprieties are always conserved. These assumptions are confirmed by the following proofs which are:

## 5 DESIGN OF THE $H_\infty$ CONTROLLER

In this section, the feedback part of the 2-DOF-MGPC controller is designed to meet robust stability and disturbance rejection requirements in a manner similar to the one Degree-Of-Freedom loop-shaping design procedure. Thereby, the robust  $H_\infty$  control design is used to achieve the better robustness properties. Its generalized feedback control system is given as [18-19],

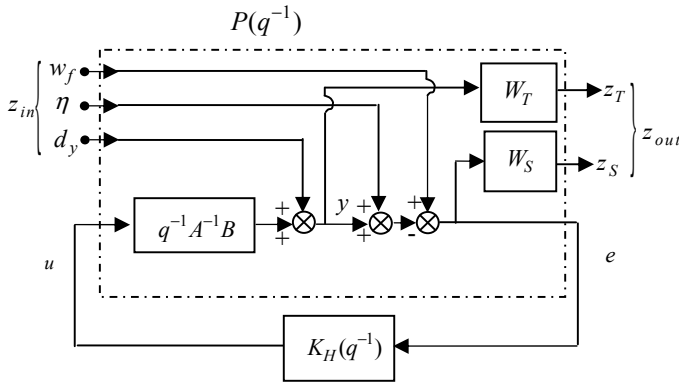


Figure 4.  $H_\infty$  generalized feedback control system

Where  $z_{in}$  and  $z_{out}$  are respectively the exogenous input and exogenous output of the generalized plant  $P(q^{-1})$ .  $w_f$  denotes the filtering set-point reference. The robust stabilizing  $H_\infty$  controller called  $K_H(q^{-1})$  is determined from minimizing the  $H_\infty$ -norm of the weighted- mixed sensitivity problem (14) using the *DHINFLMI* function of the *MATLAB* software [22]. This optimization problem is defined as [23],

$$\min_{K_H \in \mathcal{RH}_\infty} \left\| \begin{bmatrix} W_T(q^{-1})S_{c_0}(q^{-1}) \\ W_S(q^{-1})S_{d_0}(q^{-1}) \end{bmatrix} \right\|_\infty \Leftrightarrow \min_{K_H \in \mathcal{RH}_\infty} \max_{\omega \in [0, \frac{\pi}{T_e}]} \left\{ \overline{\sigma} \begin{bmatrix} W_T(\omega)S_{c_0}(\omega) \\ W_S(\omega)S_{d_0}(\omega) \end{bmatrix} \right\} \quad (14)$$

This is equivalent to the numerical inequality [23],

$$\left\| \begin{bmatrix} W_T(\omega)S_{c_0}(\omega) \\ W_S(\omega)S_{d_0}(\omega) \end{bmatrix} \right\|_\infty \leq \gamma \quad (15)$$

Where  $T_e$  and  $\gamma$  are respectively, the sampling time and the better  $H_\infty$  performance level.  $W_S$  and  $W_T$  are respectively, the weighting matrices that penalize the error and the controlled output [33-34].

$S_{d_H} = (I_{m \times m} + GK_H)^{-1}$  and  $S_{c_H} = GK_H(I_{m \times m} + GK_H)^{-1}$  are the direct sensitivity and the complementary sensitivity matrices that given by the  $H_\infty$  controller.

The objective here is to determine the Youla matrix by direct comparison between the sensitivity matrices provided by the controller  $H$  and those provided by the robustified predictive controller.

To achieve this goal, it suffices to ensure the equality of the two direct sensitivities (or complementary) of the two previous controllers, in fact to highlight the ideal matrix of Youla  $Q_{ideal}(q^{-1})$ . It is then necessary to satisfy the condition below:

$$S_{c_1}(z^{-1}) = S_{c_H}(z^{-1}) \quad (16)$$

The polynomial matrix of the *Q*-parameter is then determined as follow:

$$Q = R_0(S_{c_0}^{-1}S_{c_H} - I_{m \times m})(\Delta A)^{-1} \quad (17)$$

## 6 SIMULATION RESULTS AND DISCUSSION

The process to be controlled presents the Permanent Magnet Synchronous Machine. In order to obtain a simpler formulation and to reduce the complexity of the machine model, the establishment of its mathematic model will be developed on the basis of the assumptions that:

- The motor has a symmetrical unsaturated reinforcement, the clean and mutual inductances are independent of the currents flowing in the different windings.
- The distribution of electromotive forces along the gap is assumed to be sinusoidal.
- The losses of iron and the damping effect are neglected.
- The permeability of magnets is considered close to that of air.

The excitation being carried out by a permanent magnet, so that the excitation flux is considered constant.

In addition, the magnet is considered as a winding with no resistor or inductance proper and mutual, but as a flux source [24].

Equations of tensions and flux:

Three-phase voltages, stator flows and currents are written with the following vector notations  $V_s$ ,  $I_s$  and  $\Phi_s$  respectively.

The voltage equation in the stator frame is written [25] ; [26].

$$V_s = R_s I_s + \frac{d}{dt} \Phi_s \quad (18)$$

The stator and rotor flows have for expression:

$$\Phi_s = L_{ss} I_s + \Phi_f \quad (19)$$

The electrical equations in the park plan:

$$\begin{cases} V_{ds} = R_s I_{ds} + L_{ds} \frac{d}{dt} I_{ds} - \omega L_{qs} I_{qs} \\ V_{qs} = R_s I_{qs} + L_{qs} \frac{d}{dt} I_{qs} + \omega L_{ds} I_{ds} + \omega \Phi_f \end{cases} \quad (20)$$

Expression of power and electromagnetic torque in the park plan

$$C_e = \frac{2}{3} p [L_{ds} - L_{qs}] I_{ds} I_{qs} + \Phi_f I_{qs} \quad (21)$$

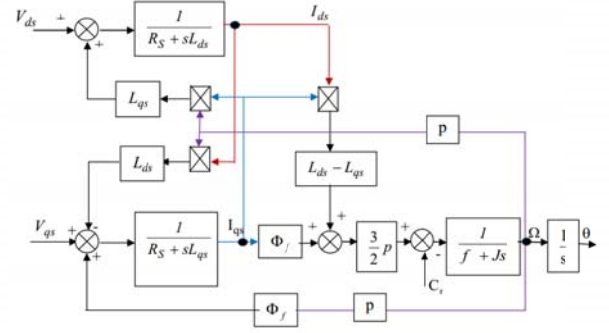
The equation of motion of the machine is:

$$C_e - C_r - f \Omega = J \frac{d\Omega}{dt} \quad (22)$$

According to equations (20), (21) and (22), we obtain the following system of equations

$$\begin{cases} \frac{d}{dt} I_{ds} = \frac{1}{L_{ds}} (V_{ds} - R_s I_{ds} + \omega L_{qs} I_{qs}) \\ \frac{d}{dt} I_{qs} = \frac{1}{L_{qs}} (V_{qs} - R_s I_{qs} + L_{ds} \omega I_{ds} + \omega \Phi_f) \\ C_e = \frac{2}{3} p [(L_{ds} - L_{qs}) I_{ds} I_{qs} + \Phi_f I_{qs}] \\ C_e - C_r - f \Omega = J \frac{d\Omega}{dt} \end{cases} \quad (23)$$

As can be seen in Figure 5 [27]:



**Figure 5. Block diagram of a voltage-fed PMSM**

According to equations (23), control the speed of the PMSM, since the transfer function of the machine can be represented in the continuous plane by the following transfer:

$$G_c(s) = \frac{(200 * Kt * s + 40 * Kt)}{J * L * s^3 + (B * L + J * (200 * Kn + R_s)) * s^2 + (B * (200 * Kn + R_s) + 40 * Kn * J + Kw * Kt) * s + 40 * Kn * B}$$

Knowing that the parameters of the PMSM are given as follows:

$$R_s = 1.4; L = L_{ds} = L_{qs} = 0.0066; J = 0.00156;$$

$$B = 0.00038818; P = 6; \phi_f = 0.175; Kw = L;$$

$$Kt = \frac{3}{2} * P * \phi_f; Kn = R_s;$$

So,

$$G_c(s) = \frac{3.144e^{07}s + 6.287e^{06}}{s^3 + 6.921e^{05}s^2 + 2.244e^{05}s + 1.722e^{04}}$$

The discretization of the transfer function gives:

$$G_d(q) = \frac{0.001754 q^3 + 0.001754 q^2 - 0.001754 q - 0.001754}{q^3 - 1.07 q^2 - 0.8605 q + 0.9302}$$

The initial GPC is synthesized with the parameters:

$$N1 = 1, N2 = 15, Nu = 2, \lambda = 0.005$$

$$R = 549.8 - 499.9 q^{-1} - 472.2 q^{-2} + 436.2 q^{-3}$$

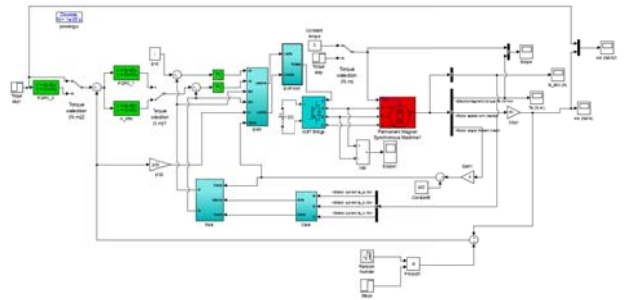
$$S = 1 - 0.4468 q^{-1} - 1.515 q^{-2} - 0.8223 q^{-3}$$

$$\begin{aligned} T = & 1.526 + 1.441 q^{-1} + 1.357 q^{-2} + 1.272 q^{-3} \\ & + 1.188 q^{-4} + 1.104 q^{-5} + 1.02 q^{-6} \\ & + 0.9346 q^{-7} + 0.8507 q^{-8} + 0.7657 q^{-9} \\ & + 0.6819 q^{-10} + 0.5967 q^{-11} \\ & + 0.513 q^{-12} + 0.4277 q^{-13} + 0.187 q^{-14} \end{aligned}$$

The weights of the system is given by

$$W_s = \frac{100 s + 1.2 e^{04}}{120 s + 0.0012} \text{ and } W_T = \frac{8.333e^{06} s + 6.944 e^{09}}{833.3 s + 8.333e^{09}}$$

Figure 6 shows the block diagram of speed regulation of the PMSM by two GPC and  $H_\infty$  type regulators, and the currents by PI type regulators.



**Figure 6. block diagram of speed regulation of the PMSM**

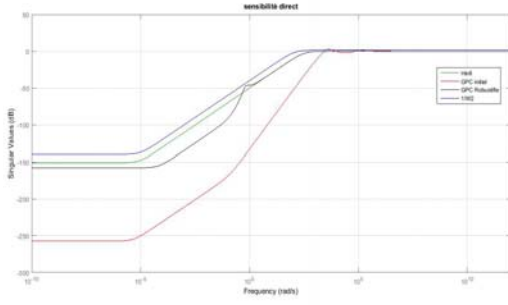


Figure 7. The direct sensitivity of the initial GPC ,  $H_\infty$  and GPC robustifie

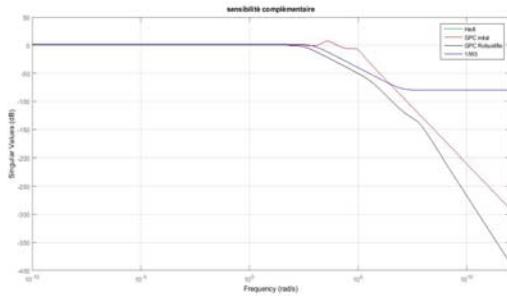


Figure 8. The complementary sensitivity of the initial GPC ,  $H_\infty$  and GPC robustifie

As shown in figure 7 and figure 8, the proposed  $Q$ -parameter ensures a perfect match between the maximal singular values of both sensitivities that given by the robustified  $GPC$  controller and those given by the  $H_\infty$  one. Therefore, the obtained robustness proprieties of initial  $GPC$  controller are modified by those given by the  $H_\infty$  controller. Moreover, according to figure 7, the maximal singular values of direct sensitivity matrices that given by three previous controllers are bounded by its upper-bound,  $1 / \overline{\sigma}[W_s(\omega)]$  at all frequencies. Similarly, the better Nominal Performances are provided by initial  $2-DOF-MGPC$  controller. However, the  $\overline{\sigma}[S_c(\omega)]$  exceeds its upper-bound,  $1 / \overline{\sigma}[W_r(\omega)]$  at some frequencies except, which is illustrated by figure 8. Thus, this result can be explained in the time domain by a higher sensitivity of the feedback control system to the effect of the measurement-noises and high frequency neglected dynamic. Figure 8 also show that the  $Q$ -parameter has the ability of decreasing slope the  $\overline{\sigma}[S_c(\omega)]$  at higher frequencies. Hence, the maximal singular values, which are provided by robustified  $GPC$  controller, are reduced at frequencies beyond the system bandwidth in order to secure robustness at high frequency range.

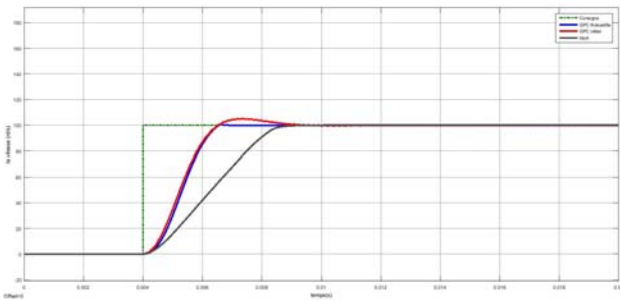


Figure 9. Comparative simulation results between the three controllers during a no-load start for a setpoint of 100 r/s.

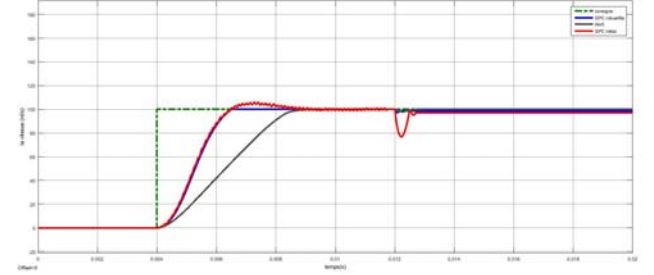


Figure 10. The results of the comparative simulation between the three correctors during a load start at  $t = 0.012s$  for a speed reference of 100 r/s

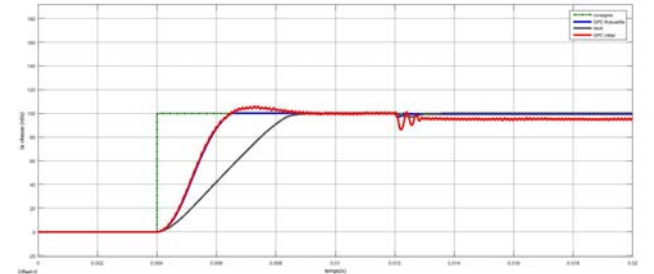


Figure 11. Comparative simulation results between the three correctors during a no-load start-up start with a noise at  $t = 0.012 s$  for a speed reference of 100 r/s.

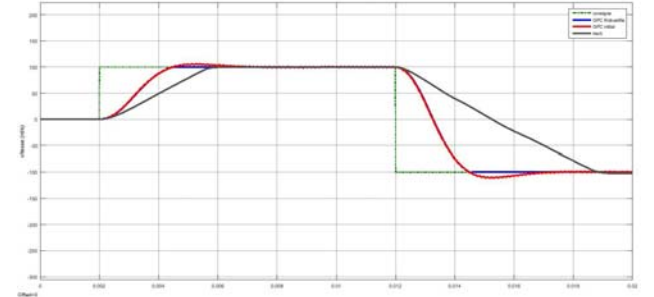
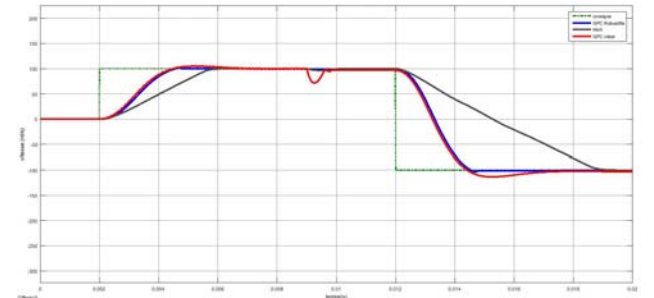
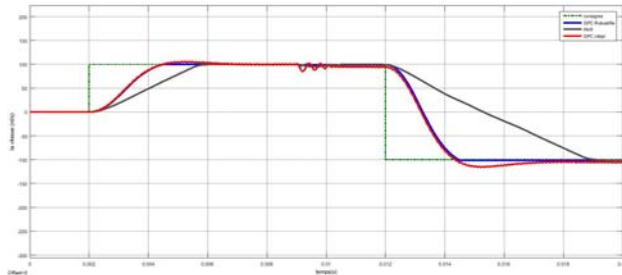


Figure 12. Comparative simulation results between the three correctors during a no-load start for a speed reference of 100 r/s with set point inversion -100 r/s.



**Figure 13. Comparative simulation results between the three correctors during a load start at  $t = 0.009s$  for a speed reference of 100 r/s with the inversion of the setpoint (-100 r/s).**



**Figure 14. Comparative simulation results between the three correctors during a vacuum start with a noise at  $t = 0.009s$  for a speed reference of 100r/s with the inversion of the setpoint (-100 r/s).**

According to the figures 9 to 14, it can be seen that the noise has an effect on the speed controlled by the initial GPC corrector in the event of a noisy signal and loss of speed, on the other hand the corrective GPC corrector and the Hinf corrector, they are robust against noise and uncertainty.

It is easy to confirm that the proposed Q parameter improves the robustness of the stability of the initial GPC controller where the measurement noises are completely rejected in the steady state. Otherwise, the best time performance is provided by a robustified GPC controller. It is explained by a fast reference tracking dynamics under transient conditions, fast rejection of disturbances and good attenuation of the effect of noise measured. These performances are guaranteed by a regular control signal compared to those given by the robustified and the initial GPC controllers.

## 7 CONCLUSION

In this paper, we examine the application of three regulators, on a synchronous machine with permanent magnets (PMSM), the latter was modeled by the transformation of Parc. The three controllers are GPC-initial, Hinf and GPC-Robustified. The simulation results obtained show that GPC-robustified by specifying the Youla parameters using several closed-loop specifications provides a very satisfactory combination of trade-offs (robustness / performance), although there are disturbances and noise. Different tests were performed or the results of the simulation showed that GPC-robustified was more powerful than GPC-initial and L'Hinf alone.

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